Stationary entanglement between macroscopic mechanical oscillators

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Abstract. We show that the optomechanical coupling between an optical cavity mode and two movable cavity mirrors is able to entangle two different macroscopic oscillation modes of the mirrors. This continuous variable entanglement is maintained by the light bouncing between the mirrors and is robust against thermal noise. In fact, it could be experimentally demonstrated using present technology.

PACS. 03.65.Ud Entanglement and quantum nonlocality (e.g. EPR paradox, Bell's inequalities, GHZ states, etc.) – 42.50.Vk Mechanical effects of light on atoms, molecules, electrons, and ions – 03.65.Yz Decoherence; open systems; quantum statistical methods

1 Introduction

Entanglement is the most characteristic trait of quantum mechanics [1]. An entangled state of a system consisting of two subsystems cannot be described as a product (or a statistical mixtures of products) of the quantum states of the two subsystems. In such a state, the system is inseparable and each component does not have properties independent of the other components. The nonlocal character of entangled states is at the basis of many paradoxes [2], and of the deep difference between the quantum and the classical world. The fundamental role of entanglement has been reemphasized in recent years after the discovery that it represents an unvaluable resource for quantum information processing [3]. In fact, entanglement is at the basis of secure quantum key distribution schemes [4], of quantum teleportation [5], and of the speed-up provided by some quantum algorithms [6]. It is generally believed that entanglement can be found only in situations involving a small number of microscopic particles. For example, a given amount of entanglement is present between two different spins in the thermal equilibrium state of a system of many interacting spins (the so-called thermal or natural entanglement [7]). However, for quantum information processing, it is the deterministic generation and manipulation of entanglement which is of paramount importance, and in these last years a number of impressive experiments has demonstrated the controlled generation of entangled states of two [8], three [9] and four [10] particles. Moreover, since entanglement is one of the distinguishing features of the quantum world, it is also fundamental to understand how far it can be extended into the macroscopic domain. This is important not only to better es-

tablish how the macroscopic classical world emerges from the microscopic one ruled by quantum mechanics [11], but also for application purposes. For example, entangled spin-squeezed states of atomic samples are known to improve the precision of frequency measurements [12], and the accuracy improves with increasing number of entangled atoms. A related question is to establish if and how two macroscopic degrees of freedom of two different objects can be entangled. With this respect, a striking achievement has been recently shown in [13], where the entanglement between the spin states of two separated Cs gas samples containing about 10^{12} atoms has been demonstrated. At the same time we proposed a feasible experiment [14] in which even a more macroscopic entanglement between the oscillating modes of two mirrors with an effective mass of some milligrams can be generated by the radiation pressure of the light bouncing between them (see also [15] for a different and extremely idealized model for the preparation of motion entangled states of two cavity mirrors). The continuous variable entanglement between two mechanical modes could be used to improve the detection of weak classical forces in optomechanical devices as atomic force microscopes or gravitational wave detectors [16,17].

In this paper we analyze in more detail and further develop the proposal of [14]. In fact, reference [14] restricted to the case of identical cavity mirrors, *i.e.*, considered, for each mirror, a single oscillation mode with identical effective mass, optomechanical coupling, damping rate and, above all, identical resonance frequency. However, [14] showed that the entanglement is present only within a small bandwidth around the mechanical resonance, and since in practice two mirrors are never exactly identical, it is important to establish the conditions under which

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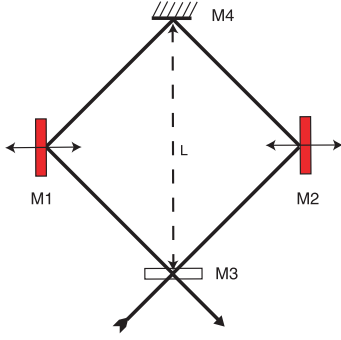
Fig. 1. Schematic description of the system under study. L, being the equilibrium distance between the movable mirrors M1, M2, is assumed to also be the distance between the fixed mirrors M3, M4. The mirror M3 represents the input-output port of the cavity.

entanglement can be generated between two mechanical modes with different resonance frequencies, and its dependence on the frequency mismatch.

In Section 2 we describe the optomechanical system under study in terms of quantum Langevin equations. In Section 3 we solve the dynamics of the system in the frequency domain, and then we characterize in detail the entanglement between the two mirrors. Section 4 is for concluding remarks.

2 The system

We consider an optical ring cavity in which two perfectly reflecting mirrors can both oscillate under the effect of the radiation pressure force (see Fig. 1). The motion of each mirror is the result of the excitation of many oscillation modes, either external [18,19] or internal [20,21]. The former is important for suspended mirrors since the excitation of pendulum modes of the suspension system leads to global displacements of the mirror. The latter corresponds to deformations of the mirror surface due to the excitation of internal acoustic modes of the substrate. These various degrees of freedom have however different resonance frequencies and one can select the mechanical response of a single particular mode by using a bandpass filter in the detection circuit [22]. For this reason we shall consider a single mechanical mode for each mirror, which will be therefore described as a simple harmonic oscillator. Since we shall consider two mirrors with similar design, the two modes will be characterized by different, but quite close, values for the frequencies, Ω_1 and Ω_2, and for the effective masses, m_1 and m_2.

The optomechanical coupling between the mirrors and the cavity field is realized by the radiation pressure. The electromagnetic field exerts a force on a movable mirror which is proportional to the intensity of the field, which, at the same time, is phase-shifted by a quantity proportional to the the mirror displacement from the equilibrium position. In the adiabatic limit in which the mirror frequency is much smaller than the cavity free spectral range $c/(2\sqrt{2}L)$ (L is the diagonal of the square optical path in the cavity,

see Fig. 1), one can focus on one cavity mode only (with annihilation operator b and frequency ω_b), because photon scattering into other modes can be neglected [23]. One gets the following Hamiltonian [24]

$$\mathcal{H} = \hbar\omega_b b^\dagger b + \sum_{i=1}^{2} \frac{\hbar\Omega_i}{2}\left(p_i^2 + q_i^2\right) - \hbar b^\dagger b \sum_{j=1}^{2}(-1)^j G_j q_j$$
$$+ i\hbar\sqrt{\gamma_b}\left(\beta^{\text{in}}e^{-i\omega_{b0}t}b^\dagger - \beta^{\text{in}*}e^{i\omega_{b0}t}b\right), \quad (1)$$

where q_i and p_i are the dimensionless position and momentum operators of the mirrors with $[q_i, p_j] = i\delta_{ij}$, $G_j = (\omega_b/L)\sqrt{\hbar/m_j\Omega_j}$ ($j = 1, 2$) are the optomechanical coupling constants, and the last terms in equation (1) describe the laser driving the cavity mode, characterized by a frequency ω_{b0} and a power $P_b^{\text{in}} = \hbar\omega_{b0}|\beta^{\text{in}}|^2$ (γ_b is the cavity mode linewidth).

A detailed analysis of the problem, however, must include photon losses, and the thermal noise on the mirrors. It means that the interaction of the optical mode with its reservoir and the effect of thermal fluctuations on the two mirrors, not considered in Hamiltonian (1), must be added. This can be accomplished in the standard way [25,26]. We neglect instead all the technical sources of noise, *i.e.*, we shall assume that the driving laser is stabilized in intensity and frequency, also because recent experiments have shown that classical laser noise can be made negligible in the relevant frequency range [19,20]. The full quantum dynamics of the system can be exactly described by the following nonlinear Langevin equations (in the interaction picture with respect to $\hbar\omega_b b^\dagger b$)

$$\dot{b} = i(\omega_{b0} - \omega_b)b - ib(G_1 q_1 - G_2 q_2)$$
$$- \frac{\gamma_b}{2}b + \sqrt{\gamma_b}\left(b^{\text{in}} + \beta^{\text{in}}\right),$$
$$\dot{q}_j = \Omega_j p_j,$$
$$\dot{p}_j = -\Omega_j q_j + (-)^j G_j b^\dagger b - \Gamma_j p_j + \xi_j, \quad (2)$$

where Γ_j ($j = 1, 2$) are the mechanical damping rates of the mechanical modes, $b^{\text{in}}(t)$ represent the vacuum white noise operator at the cavity input [25], and the Langevin noise operators for the quantum Brownian motion of the mirrors are $\xi_j(t)$. The non-vanishing noise correlations are

$$\langle b^{\text{in}}(t)b^{\text{in}\dagger}(t')\rangle = \delta(t - t'), \quad (3)$$
$$\langle \xi_j(t)\xi_k(t')\rangle = \delta_{j,k}\int_0^\infty d\omega \frac{\Gamma_j\omega}{2\Omega_j}$$
$$\times \left[\coth\left(\frac{\hbar\omega}{2k_{\text{B}}T}\right)\cos\left[\omega(t - t')\right] - i\sin\left[\omega(t - t')\right]\right],$$

where k_{B} is the Boltzmann constant and T the equilibrium temperature (the two mirrors are considered in equilibrium with their respective bath at the same temperature). Notice that the used approach for the Brownian motion is quantum mechanical consistent at every temperature [26].

We consider the situation when the driving field is very intense. Under this condition, the system is characterized by a semiclassical steady state with the internal cavity

mode in a coherent state $|\beta\rangle$, and a displaced equilibrium position for the mirrors. The steady state values are obtained by taking the expectation values of equations (2), factorizing them and setting all the time derivatives to zero. One gets

$$\langle q_j\rangle_{ss} = (-)^j G_j|\beta|^2/\Omega_j, \qquad \langle p_j\rangle_{ss} = 0,$$
$$\beta \equiv \langle b\rangle_{ss} = \sqrt{\gamma_b}\beta^{\text{in}}/\left(\gamma_b/2 - \mathrm{i}\Delta_b\right), \qquad (4)$$

where $\Delta_b \equiv \omega_{b0} - \omega_b - G_1\langle q_1\rangle_{ss} + G_2\langle q_2\rangle_{ss}$, is the cavity mode detuning.

Under these semiclassical conditions, the dynamics is well described by linearizing equations (2) around the steady state. If we now use the same symbols for the operators describing the quantum fluctuations around the steady state, we get the following linearized quantum Langevin equations

$$\dot{b} = \mathrm{i}\Delta_b b - \mathrm{i}\beta(G_1 q_1 - G_2 q_2) - \frac{\gamma_b}{2}b + \sqrt{\gamma_b}b^{\text{in}}, \quad \dot{q}_j = \Omega_j p_j,$$
$$\dot{p}_j = -\Omega_j q_j + (-)^j G_j(\beta^* b + \beta b^\dagger) - \Gamma_j p_j + \xi_j. \qquad (5)$$

3 Entanglement characterization

The time evolution of the system can be easily obtained by solving the linear quantum Langevin equations (5). However, as it happens in quantum optics for squeezing (see for example [25]), it is more convenient to study the system dynamics in the frequency domain. In fact, it is possible that, due to the effect of damping, and thermal and quantum noise, the two mechanical modes of the mirrors are never entangled in time, *i.e.*, there is no time instant in which the reduced state of the two mechanical modes is entangled, unless appropriate (but difficult to prepare) initial conditions of the whole system are considered. Entanglement can be instead always present at a given frequency. In fact, the two mirrors constitute, for each frequency, a continuous variable bipartite system which, in a given frequency bandwidth, can be in an entangled state. The Fourier analysis refers to the quantum fluctuations around the semiclassical steady state discussed in the preceding section, and the eventual entanglement found at a given frequency would refer to a stationary state of the corresponding spectral modes, maintained by the radiation mode, and which decays only when the radiation is turned off. The spectral analysis is more econvenient also because in such systems the dynamics is experimentally better studied in frequency rather than in time. The same kind of spectral analysis of the nonlocal properties of a bipartite continuous variable system has been already applied in reference [27] which demonstrated the EPR nonlocality between two optical beams of a nondegenerate parametric amplifier, following the suggestion of [28].

Performing the Fourier transform of equations (5), one easily gets for the mechanical modes operators ($j = 1, 2$)

$$q_j(\omega) = \mathcal{B}_j(\omega)b^{\text{in}}(\omega) + \mathcal{B}_j^*(-\omega)b^{\text{in}\,\dagger}(-\omega)$$
$$+ \Xi_{j,1}(\omega)\xi_1(\omega) + \Xi_{j,2}(\omega)\xi_2(\omega) \qquad (6)$$
$$p_j(\omega) = -\mathrm{i}\frac{\omega}{\Omega_j}q_j(\omega), \qquad (7)$$

where

$$\mathcal{B}_j(\omega) = (-)^j \frac{1}{\mathcal{D}(\omega)}\left[\frac{1}{\Omega_{3-j}\chi_{3-j}(\omega)}\right]\left[\frac{\sqrt{\gamma_b}G_j\beta^*}{\gamma_b/2 - \mathrm{i}\left(\Delta_b + \omega\right)}\right], \qquad (8a)$$

$$\Xi_{j,k}(\omega) = \frac{1}{\mathcal{D}(\omega)}\left\{\frac{1}{\Omega_{3-j}\chi_{3-j}(\omega)}\delta_{j,k} - \mathrm{i}G_{3-j}G_{3-k}|\beta|^2\right.$$
$$\times\left.\left[\frac{1}{\gamma_b/2 - \mathrm{i}\left(\Delta_b + \omega\right)} - \frac{1}{\gamma_b/2 + \mathrm{i}\left(\Delta_b - \omega\right)}\right]\right\}, \qquad (8b)$$

$$\mathcal{D}(\omega) = \frac{1}{\Omega_1\Omega_2\chi_1(\omega)\chi_2(\omega)} - \mathrm{i}|\beta|^2\left[\frac{G_1^2}{\Omega_2\chi_2(\omega)} + \frac{G_2^2}{\Omega_1\chi_1(\omega)}\right]$$
$$\times\left[\frac{1}{\gamma_b/2 - \mathrm{i}\left(\Delta_b + \omega\right)} - \frac{1}{\gamma_b/2 + \mathrm{i}\left(\Delta_b - \omega\right)}\right], \qquad (8c)$$

and $\chi_j(\omega) = [\Omega_j^2 - \omega^2 - \mathrm{i}\omega\Gamma_j]^{-1}$ is the mechanical susceptibility of mode j. Notice that $\Xi_{j,k}^*(\omega) = \Xi_{j,k}(-\omega)$ and $\mathcal{D}^*(\omega) = \mathcal{D}(-\omega)$, but $\mathcal{B}^*(\omega) \neq \mathcal{B}(-\omega)$.

The simplest way to establish the parameter region where the oscillation modes of the two cavity mirrors are entangled is to use one of the sufficient criteria for entanglement of continuous variable systems already existing in the literature. These criteria are inequalities which have to be satisfied by the product [14,29,30] or the sum [31,32] of variances of appropriate linear combinations of the rescaled position and momentum operators of the continuous variable systems. These criteria are usually formulated in terms of Heisenberg operators at the same time instant, satisfying the usual commutation relations $[q_j(t), p_k(t)] = \mathrm{i}\delta_{jk}$ [29,31,32], but they can be adapted to their Fourier transform, as long as the commutators between the frequency-dependent continuous variable operators are still a *c*-number [14]. This condition is satisfied in the present case thanks to the linearity of the Fourier transform and to the linear dynamics of the fluctuations (see Eqs. (5)), implying that the commutators are always *c*-number frequency-dependent functions.

The paradigmatic entangled state for continuous variable systems is the state considered by Einstein, Podolski and Rosen in their famous paper [2], *i.e.*, the simultaneous eigenstate of the relative distance $q_1 - q_2$ and of the total momentum $p_1 + p_2$. In an entangled state of this kind, the variances of these two operators are both small and it is therefore natural to use them. Defining $u = q_1 - q_2$ and $v = p_1 + p_2$, an inseparability criterion for the sum of the variances in the case of arbitrary *c*-number commutators is [31]

$$\langle(\Delta u)^2\rangle + \langle(\Delta v)^2\rangle < 2|\langle[q_1, p_1]\rangle|^2, \qquad (9)$$

while that for the product of variances is [14,29,30]

$$\langle(\Delta u)^2\rangle\langle(\Delta v)^2\rangle < |\langle[q_1, p_1]\rangle|^2. \qquad (10)$$

It is easy to see that the condition (9) implies condition (10), which means that the product criterion (10) is easier to satisfy, and for this reason we shall consider only the latter from now on. Furthermore, the product
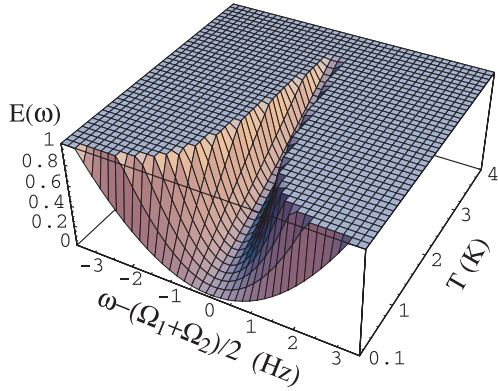
Fig. 2. Degree of entanglement $E(\omega)$ of equation (14) *versus* frequency and temperature T, in the case of equal mechanical resonance frequencies, $\Omega_1 = \Omega_2 = 1$ MHz. The plot has been cut at $E(\omega) = 1$. The other parameter values are in the text.
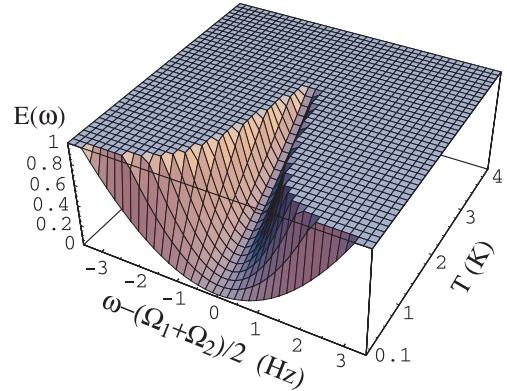
Fig. 3. Degree of entanglement $E(\omega)$ of equation (14) *versus* frequency and temperature T, in the case of a mechanical frequency mismatch $\Omega_2 - \Omega_1 = 10$ Hz. The plot has been cut at $E(\omega) = 1$. The other parameter values are in the text.

criterion (10) allows us to establish a connection with references [28], which showed that when the inequality

$$\langle(\Delta u)^2\rangle\langle(\Delta v)^2\rangle < \frac{1}{4}|\langle[q_1, p_1]\rangle|^2, \qquad (11)$$

is satisfied, an EPR-like paradox arises [2,33], based on the inconsistency between quantum mechanics and local realism, which has been then experimentally confirmed in [27]. The sufficient condition for inseparability of equation (10) is weaker than condition (11), but this is not surprising, since entangled states are only a necessary condition for the realization of an EPR-like paradox (see however the recent paper [30] where it is shown that the weaker inseparability sufficient condition (10) can be considered as a marker of the existence of generalized, weaker, EPR correlations).

In order to apply the inseparability criterion (10) in the frequency domain, we have to make the frequency dependent operators $q_j(\omega)$ and $p_j(\omega)$ Hermitian, *i.e.*, to consider the Hermitian component

$$\mathcal{R}_{\mathcal{O}}(\omega) = \frac{\mathcal{O}(\omega) + \mathcal{O}(-\omega)}{2} \qquad (12)$$

for any operator $\mathcal{O}(\omega)$. Using the fact that $\langle q_j(\omega)\rangle = \langle p_j(\omega)\rangle = 0$, $j = 1, 2$ and $\forall\omega$ because they are associated to fluctuations around the semiclassical steady state, equation (10) therefore becomes

$$\langle\mathcal{R}_{q_1-q_2}^2\rangle\langle\mathcal{R}_{p_1+p_2}^2\rangle < |\langle[\mathcal{R}_{q_1}, \mathcal{R}_{p_1}]\rangle|^2, \qquad (13)$$

which suggests the following definition of degree of entanglement for the mechanical oscillation modes at frequency ω of the two cavity mirrors [14]

$$E(\omega) = \frac{\langle\mathcal{R}_{q_1-q_2}^2\rangle\langle\mathcal{R}_{p_1+p_2}^2\rangle}{|\langle[\mathcal{R}_{q_1}, \mathcal{R}_{p_1}]\rangle|^2}, \qquad (14)$$

which is a marker of entanglement whenever $E(\omega) < 1$.

Using equations (8) it is possible to derive the analytic expression of $E(\omega)$, which is however very cumbersome.

The two variances in the numerator of (14) are

$$\langle\mathcal{R}_{q_1-q_2}^2\rangle = \frac{1}{4}\left\{|\mathcal{B}_1(\omega) - \mathcal{B}_2(\omega)|^2 + |\mathcal{B}_1(-\omega) - \mathcal{B}_2(-\omega)|^2\right.$$
$$+ \mathcal{N}_1(\omega)|\Xi_{1,1}(\omega) - \Xi_{2,1}(\omega)|^2$$
$$\left. + \mathcal{N}_2(\omega)|\Xi_{1,2}(\omega) - \Xi_{2,2}(\omega)|^2\right\}, \qquad (15)$$

$$\langle\mathcal{R}_{p_1+p_2}^2\rangle = \frac{1}{4}\left(\frac{\omega}{\Omega_1}\right)^2\left\{|\mathcal{B}_1(\omega)|^2 + |\mathcal{B}_1(-\omega)|^2\right.$$
$$\left. + \mathcal{N}_1(\omega)\left[|\Xi_{1,1}(\omega)|^2 + |\Xi_{2,1}(\omega)|^2\right]\right\}$$
$$+ \frac{1}{4}\left(\frac{\omega}{\Omega_2}\right)^2\left\{|\mathcal{B}_2(\omega)|^2 + |\mathcal{B}_2(-\omega)|^2\right.$$
$$\left. + \mathcal{N}_2(\omega)\left[|\Xi_{1,2}(\omega)|^2 + |\Xi_{2,2}(\omega)|^2\right]\right\}$$
$$+ \frac{1}{4}\left(\frac{\omega^2}{\Omega_1\Omega_2}\right)\left\{\mathcal{B}_1(\omega)\mathcal{B}_2^*(\omega) + \mathcal{B}_1(-\omega)\mathcal{B}_2^*(-\omega)\right.$$
$$+ \mathcal{B}_1^*(\omega)\mathcal{B}_2(\omega) + \mathcal{B}_1^*(-\omega)\mathcal{B}_2(-\omega)$$
$$+ \mathcal{N}_1(\omega)\left[\Xi_{1,1}(\omega)\Xi_{2,1}(-\omega) + \Xi_{1,1}(-\omega)\Xi_{2,1}(\omega)\right]$$
$$+ \mathcal{N}_2(\omega)\left[\Xi_{1,2}(\omega)\Xi_{2,2}(-\omega)\right.$$
$$\left.\left. + \Xi_{1,2}(-\omega)\Xi_{2,2}(\omega)\right]\right\}, \qquad (16)$$

with $\mathcal{N}_j(\omega) = \omega(\Gamma_j/\Omega_j)\coth(\hbar\omega/2k_BT)$, while the commutator in the denominator of (14) is given by

$$\langle[\mathcal{R}_{q_1}, \mathcal{R}_{p_1}]\rangle = \frac{i}{2}\frac{\omega}{\Omega_1}\left\{|\mathcal{B}_1(\omega)|^2 - |\mathcal{B}_1(-\omega)|^2\right.$$
$$\left. - \omega\frac{\Gamma_1}{\Omega_1}\left[|\Xi_{1,1}(\omega)|^2 + |\Xi_{1,2}(\omega)|^2\right]\right\}. \qquad (17)$$

In Figures 2–4 we have studied the behaviour of $E(\omega)$ as a function of frequency and temperature, for different values of the difference between the two resonance frequencies of the mechanical modes, $\Omega_1 - \Omega_2$. This is an important parameter because we have seen in [14] that in the case of identical mirrors, the two mechanical modes
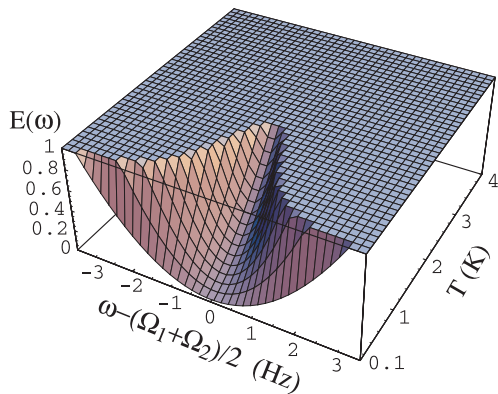
Fig. 4. Degree of entanglement $E(\omega)$ of equation (14) *versus* frequency and temperature T, in the case of a mechanical frequency mismatch $\Omega_2 - \Omega_1 = 20$ Hz. The plot has been cut at $E(\omega) = 1$. The other parameter values are in the text.

are entangled only within a small bandwidth around the mechanical resonance. Since in practice the two mirrors will never be exactly identical, it is important to establish if the macroscopic entanglement is able to tolerate a certain amount of frequency mismatch. For the other parameter values we have considered an experimental situation comparable to that of references [20, 22, 34], where the studied mirror oscillation mode is a Gaussian acoustic mode. We have therefore considered a cavity driven by a laser working at $\lambda = 810$ nm and power $P_b^{\rm in} = 1$ W, with length $L = 1$ mm, detuning $\Delta_b = 6$ MHz, optical finesse $\mathcal{F} = 25\,000$, yielding a cavity decay rate $\gamma_b = 6$ MHz. The mechanical modes have been taken with effective mass $m_1 = m_2 = 23$ mg, damping rates $\Gamma_1 = \Gamma_2 = 1$ Hz, and $\Omega_1 = 1$ MHz, while we have changed the values of Ω_2 around those of Ω_1. These choices yield for the optomechanical couplings $G_1 \simeq G_2 \simeq 2.5$ Hz.

Figure 2 shows $E(\omega, T)$ for no frequency mismatch, $\Omega_1 = \Omega_2$, Figure 3 refers to the case with $\Omega_2 - \Omega_1 = 10$ Hz, and Figure 4 refers to the case with $\Omega_2 - \Omega_1 = 20$ Hz. In all cases, the region of the ω, T plane where the two mechanical modes are entangled is centered in the middle of the two mechanical resonances, *i.e.*, $E(\omega, T)$ always achieves its minimum at $\omega = (\Omega_1 + \Omega_2)/2$. The frequency bandwidth of the entanglement region rapidly decreases with increasing temperature, so that, with the chosen parameter values, entanglement disappears above $T \simeq 4$ K. As expected, the ω, T region where the two mirrors are entangled becomes smaller for increasing frequency mismatch (compare the three figures). Nonetheless these results are extremely interesting because they clearly show the possibility to entangle two *macroscopic oscillators* (with an effective mass of 23 mg) in a stationary way, using present technology. In fact, the two modes are still clearly entangled at $T = 2$ K and with $\Omega_2 - \Omega_1 = 10$ Hz (ten times larger than the width of the mechanical resonance peaks, see Fig. 3), while one is forced to go below $T = 2$ K when the frequency mismatch is equal to 20 Hz (see Fig. 4).

Differently from temperature and frequency mismatch, and as it can be seen from the involved analytical expres-

sion above, it is difficult to determine how the degree of entanglement depends upon the other parameters. It can only be verified that, as expected, entanglement improves with increasing mechanical quality factor $Q_j = \Omega_j/\Gamma_j$ and that it strongly improves with increasing the effective optomechanical coupling constant, which is given by βG_j (see Eqs. (5)). This shows that for achieving even a more macroscopic entanglement (*i.e.*, larger masses), one has to use smaller cavities and, above all, larger optical power. The fundamental importance of the effective coupling constant βG_j also helps us to show which kind of entangled state of the two mirrors is generated by the radiation pressure. In fact, when the cavity mode intensity becomes larger and larger, the optomechanical interaction tends to project the two mechanical modes onto an approximate eigenstate of $G_1 q_1 - G_2 q_2$ (see Eqs. (1, 5)), which, since in our case it is $G_1 \simeq G_2$, is essentially equivalent to the relative distance $q_1 - q_2$. The two oscillators occupy a state that, like a standard EPR state, has a very small variance of the relative distance $u = q_1 - q_2$. On the other hand, since the radiation pressure does not have analogous effects on the total momentum $v = p_1 + p_2$, the state of the mirrors does not exhibit such a small value for the variance $\langle (\Delta v)^2 \rangle$ as the standard EPR state does. Nonetheless, at large optical intensities, as shown by the product criterion of equation (10), the effect of the radiation pressure force on the relative distance is sufficient to entangle the two macroscopic oscillator modes. Moreover, as it can be seen from Figures 2–4, the degree of entanglement $E(\omega)$ lies even below 1/4 at sufficiently low temperatures, allowing therefore in principle also an experimental test of EPR nonlocality with macroscopic oscillators, on the basis of the inequality (11) of references [28].

4 Conclusions

We have shown how the optomechanical coupling realized by the radiation pressure of an optical mode of a ring cavity is able to entangle two macroscopic collective oscillation modes of two cavity mirrors. Using parameter values corresponding to already performed experiments involving an optical cavity mode coupled to an acoustic mode of the mirror (with an effective mass of many milligrams) we have shown that an appreciable entanglement is achievable at temperatures of some kelvin. This continuous variable entanglement is established at a given frequency, between the spectrally decomposed oscillation modes of the two mirrors (see also Refs. [27, 28] for an analogous spectral analysis of the nonlocal properties of the beams of a nondegenerate optical amplifier). One has a stationary entanglement, which is maintained by the strongly driven cavity mode as long as it is turned on. Using the degree of entanglement $E(\omega)$ of equation (14), suggested by the inseparability condition of equation (10), we have seen that the entanglement is more robust when the two mechanical resonance frequencies are equal (Fig. 2), but that it tolerates a resonance frequency mismatch of tens of Hz, much larger than the width of the resonance peaks. The best entanglement is always achieved in the middle of the

two mechanical resonances and the frequency bandwidth of the entanglement parameter region rapidly decreases with decreasing optomechanical coupling and increasing temperatures.

This continuous variable entanglement between two macroscopic collective degrees of freedom can be experimentally measured using for example the three-cavity scheme described in detail in [14]. In such a scheme, a ring cavity is supplemented with two other external cavities, each measuring the spectral components $q_j(\omega)$ and $p_j(\omega)$ of each mirror oscillation mode *via* homodyne detection. With these measurements, it is possible to obtain both variances $\langle \mathcal{R}^2_{q_1-q_2} \rangle$ and $\langle \mathcal{R}^2_{p_1+p_2} \rangle$. As it has been verified in [14], if the driving power of the meter cavities is much smaller than the driving power of the "entangler" cavity mode, the two additional cavities do not significantly modify the entanglement dynamics. A simplified detection scheme, involving less than three cavities is currently investigated. In fact, the homodyne detection of the entangler mode b provides direct information on the relative distance between the mirror modes $q_1 - q_2$. The measurement of the total momentum quadrature $p_1 + p_2$ could be then achieved using the result of this homodyne detection and that of the homodyne measurement of the motion of a single mirror provided by a second "meter" cavity mode. It is however possible that an even simpler detection scheme exists, using the entangler cavity mode only.

Another important aspect which has to be taken into account is that the motion of each mirror is the superposition of many oscillation modes with different resonance frequencies. We can safely verify the entanglement between the two considered oscillation modes provided that the other modes of the two mirrors are sufficiently far away in frequency so that their contribution at the analysed frequencies is negligible. Moreover, the above analysis also applies, almost unmodified, to the case when the two modes belong to the same mirror.

The possibility to prepare entangled state of two macroscopic degrees of freedom is not only conceptually important for better understanding the relation between the macroscopic world ruled by classical mechanics and the quantum mechanical microscopic substrate, but it may also prove to be useful for some applications. For example, reference [16] has showed that entangled states of the kind studied here could improve the detection of weak mechanical forces acting on the mirrors as those due to gravitational waves [35].

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